

## Mech-465 Automatic Control

### Lecture 2 Assignment

1. Solve the differential equation

$$\frac{d^2x(t)}{dt^2} + 5\frac{dx(t)}{dt} + 4x(t) = 10u(t)$$

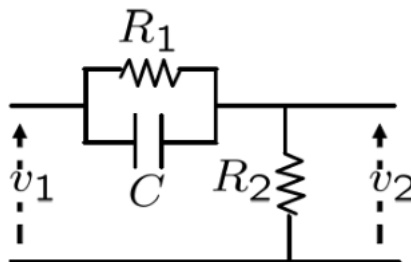
with the following initial conditions:

(a)  $x(0) = \dot{x}(0) = 0$

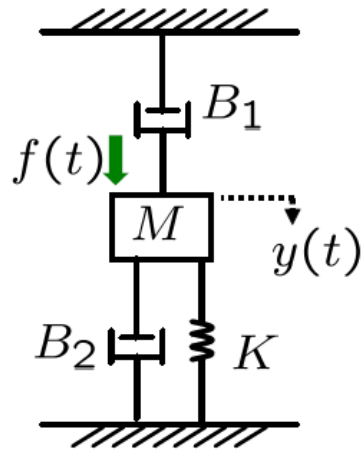
2. For the following  $G(s)$ , transform the relation  $C(s) = G(s)R(s)$  into the differential equation.

$$G(s) = \frac{60}{s^2 + 10s + 60}$$

3. For the following circuit, find the transfer function from  $V_1(s)$  to  $V_2(s)$ . If we input a constant voltage 10V, what is the steady state value of the output voltage?  
(Hint: Use  $v_1(t) = 10 \rightarrow V_1(s) = 10/s$  then find the final value theorem for  $v_2(t)$ )



4. For the following mechanical systems, obtain the transfer function from the input force  $f$  to the mass displacement  $y$



1)

$$\frac{d^2 x(t)}{dt^2} + 5 \frac{dx(t)}{dt} + 4 \cdot x(t) = 10 u(t)$$

Laplace transform

$$(s^2 X(s) - s x(0) - \dot{x}(0)) + 5(s X(s) - x(0)) + 4 X(s) = \frac{10}{s}$$

$$\Rightarrow X(s) = \frac{1}{s^2 + 5s + 4} \left( \frac{10}{s} + (s+5) x(0) + \dot{x}(0) \right) \quad \text{--- (2)}$$

$$a) x(0) = \dot{x}(0) = 0$$

$$\Rightarrow X(s) = \frac{10}{s(s^2 + 5s + 4)} = \boxed{\frac{10}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}} \quad \text{--- (1)}$$

(1)  $\times s$  to obtain A

$$\frac{10}{(s+1)(s+4)} \Big|_{s \rightarrow 0} = A + 0 + 0 \Rightarrow A = \frac{10}{4}$$

(1)  $\times (s+1)$  to obtain B:

$$\frac{10}{s(s+4)} \Big|_{s \rightarrow -1} = 0 + B + 0 \Rightarrow B = \frac{-10}{3}$$

(1)  $\times (s+4)$  to obtain C

$$\frac{10}{s(s+1)} \Big|_{s \rightarrow -4} = 0 + 0 + C \Rightarrow C = \frac{10}{-3(-4)} = \frac{10}{12} = \frac{5}{6}$$

$$X(s) = \frac{10}{s(s+1)(s+4)} = \frac{\frac{5}{4}}{\frac{1}{s}} + \frac{-\frac{10}{3}}{\frac{1}{s+1}} + \frac{\frac{5}{6}}{\frac{1}{s+4}}$$

$$\Rightarrow x(t) = \frac{5}{2} - \frac{10}{3} e^{-t} + \frac{5}{6} e^{-4t}$$

2)

$$a) G(s) = \frac{60}{s^2 + 10s + 60}$$

$$C(s) = G(s) R(s)$$

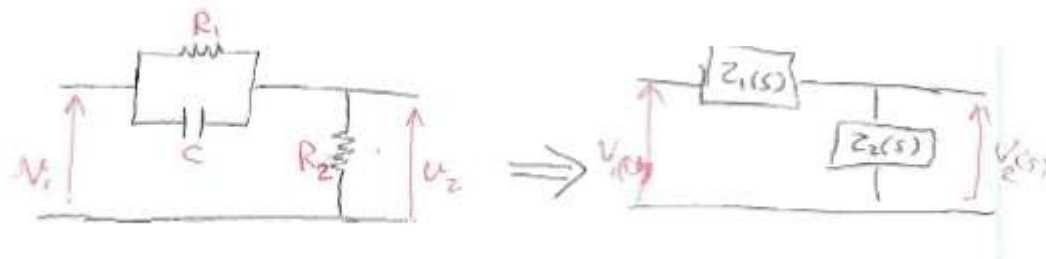
$$C(s) = \frac{60}{s^2 + 10s + 60} R(s)$$

$$(s^2 + 10s + 60) C(s) = 60 R(s)$$

$$s^2 C(s) + 10s C(s) + 60 C(s) = 60 R(s)$$

$$\ddot{c}(t) + 10\dot{c}(t) + 60c(t) = 60r(t) \quad \leftarrow \mathcal{L}^{-1}$$

3)



Using Kirchhoff's law

$$V_1(s) = Z_1(s) + Z_2(s) \quad \left( Z_1(s) = \frac{\frac{R_1}{Cs}}{R_1 + \frac{1}{Cs}} \right)$$

$$V_2(s) = Z_2(s) \quad (Z_2(s) = R_2)$$

$$\therefore TF = \frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

$$= \frac{R_2}{\frac{\frac{R_1}{Cs}}{R_1 + \frac{1}{Cs}} + R_2} = \frac{R_2}{\frac{R_1}{R_1 Cs + 1} + R_2}$$

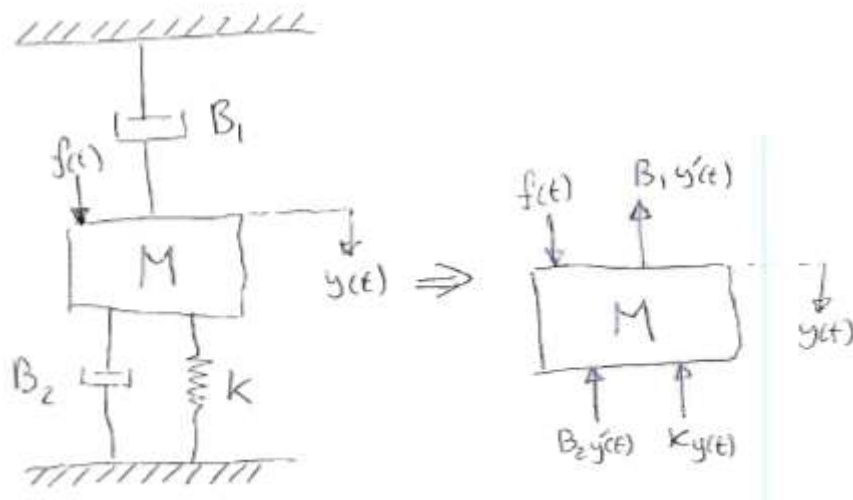
$$= \frac{R_2}{\frac{R_1 + R_2(R_1 Cs + 1)}{R_1 Cs + 1}} = \frac{R_1 R_2 Cs + R_2}{R_1 R_2 Cs + R_2 + R_1}$$

$$\text{For } V_1(s) = \frac{10}{s} \Rightarrow V_2(s) = \frac{R_1 R_2 Cs + R_2}{R_1 R_2 Cs + R_2 + R_1} \cdot \frac{10}{s}$$

$$\lim_{t \rightarrow \infty} V_2(t) = \lim_{s \rightarrow 0} s V_2(s) = \lim_{s \rightarrow 0} \cancel{s} \cdot \frac{R_1 R_2 Cs + R_2}{R_1 R_2 Cs + R_2 + R_1} \cdot \frac{10}{\cancel{s}}$$

$$= \frac{10 R_2}{R_1 + R_2}$$

4)



$$M \ddot{y}(t) = f(t) - (B_1 + B_2) \dot{y}(t) - k y(t)$$

$$M s^2 Y(s) = F(s) - (B_1 + B_2) s Y(s) - k Y(s) \quad \leftarrow \text{Laplace}$$

$$(M s^2 + (B_1 + B_2) s + k) Y(s) = F(s)$$

$$\frac{Y(s)}{F(s)} = \frac{1}{M s^2 + (B_1 + B_2) s + k}$$